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The peculiarities of sprayed liquid's thermal state change, as droplets are heated by conduction

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Abstract

The change of thermal state and phase transformation intensity of sprayed water, *n*-hexane, *n*-heptane and *n*-decane is numerically modelled in the case, as droplets are heated by conduction; the influence of the Knudsen layer is neglected; warming and evaporation of the droplets has no influence on the state of the carrying air flow. The research results prove that a peculiar change of the thermal state of sprayed liquid, irrespective of droplet's dispersivity, exists in the time scale, expressed by Fourier number. The above-mentioned change can be conveniently defined by the characteristic curves, representing the change of a droplet surface, centre, and mean mass temperatures, which are sensibly influenced by temperature of gas mixture and partial pressure of liquid vapour in it. As these characteristic curves were expressed in regards to the initial and equilibrium evaporation temperatures of liquid, the universal curves, representing the change of thermal states of the examined liquids, were obtained in the time scale, expressed by Fourier number. It is shown that liquid evaporation rate and the change of a droplet dimension can also be described by characteristic curves.

1. Introduction

Many thermal technologies are in one or another way related to liquid injection. Liquid fuel combustion in boilers, internal combustion or jet engines; evaporation of water in air conditioning systems; rapid gas cooling; thermal process regulation; surface protection processes can be good examples of such thermal technologies. Their efficiency depends on heat and mass transfer between gas and liquid droplets. Though the research of heat and mass transfer in liquid droplets have more than 100 year old history [1], they are very important and actual in our days as well. It is conditioned by the variety of boundary conditions of heat and mass transfer, occurring in two-phase flows and by deeper demand of cognition of transfer process interaction. Process interaction is influenced by many factors: unsteadiness of heat and mass transfer, which actually restricts experimental research of "the droplet" problem and requires more complicated mathematical approach; the influence of the Stefan's hydrodynamic flow on liquid evaporation rate, droplet motion dynamics and their convective heating; selectivity of radiation in gas and the peculiarities of radiant absorption in semitransparent liquid droplets, influencing temperature field's gradient in a droplet; the influence of the Knudsen layer on warming of small droplets and phase transformations on their surfaces; droplet slip in gas, which conditions convective heating intensity and determines forced circulation of liquid inside droplet; coalescence and decomposition of droplets, which may change liquid dispersity of a two-phase flow; gradual formation of a hard shell on the surface of a multicomponental liquid droplets, which conditions essential changes of droplet's thermal state and phase transformations; the mode of explosive evaporation, which makes assumptions of formation of a monodisperse-like two-phase flow, consisting of very small droplets; the peculiarities of

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Nomenclature

a	thermal diffusivity
B	Spalding transfer number
c_p	mass specific heat
D	mass diffusivity
Fo	Fourier number
g	mass flow rate
I	control time index
k_c^-	effective conductivity parameter
L	latent heat of evaporation
m	vapour mass flux
M	mass
n	number of the term in infinite sum
N	number of terms in infinite sum
Nu	Nusselt number
p	pressure
Pe	Peclet number
Pr	Prandtl number
q	heat flux
r	coordinate of a droplet
R	radius of a droplet
Re	Reynolds number
R_μ	universal gas constant
T	temperature
w	velocity

Greek symbols

η	non-dimensional coordinate
λ	thermal conductivity
μ	molecular mass
ρ	density
τ	time

Subscripts

c	convective
C	droplet centre
ch	characteristic
e	equilibrium evaporation
f	evaporation
g	gas
vg	vapour–gas mixture
i	time index in a digital scheme
I	index of control time
j	index of droplet cross-section
J	index of droplet surface
k	conductive, number of an iteration
K	number of iterations
l	liquid
m	mass average
r	radiation
R	droplet surface
s	condition of saturation
sor	source
v	vapour
0	initial state
∞	far from a droplet

Superscripts

$+$	external side of a surface
$-$	internal side of a surface

two-phase flow geometry, etc. The above-mentioned questions are very important for modern research of the “droplet” problem.

The results of early research of the “droplet” problem, when steady heat and mass transfer was examined, as droplet evaporates in low temperature surroundings, are generalized in [1]. Introduction of the Spalding transfer number [2] allowed evaluating the influence of the Stefan’s flow on phase transformations of sprayed liquid and convective heating of droplets by the methods of similarity theory. Thorough research of the “droplet” problem is presented in [3–5]. The achievements of the last decades in the field of “droplet” problem are overviewed in [6]. The intensity of heating of evaporating droplets is calculated according to the empirical expressions, constructed for non-evaporating particles, though introducing the functions of the Spalding transfer number: $Nu_\mathrm{f} = Nu \cdot f(B_T)$. The models of non-evaporating particle are thoroughly discussed in [7]. Various models, which evaluate the influence of the Stefan’s hydrodynamic flow on convective heating intensity of an evaporating droplet, are used. The most popular models include the classical model [6]: $f(B_T) = B_T^{-1} \cdot \ln(1+B_T)$, the experimental correlation model [4]: $f(B_T) = (1+B_T)^{-0.7}$, and the Abramzon–Sirignano model, which connects the above – two models [3]: $Nu_\mathrm{f} = [2 + F_T^{-1} \cdot (Nu-2)] \cdot B_T^{-1} \cdot \ln(1+B_T)$; $F_T = (1+B_T)^{0.7} B_T^{-1} \cdot \ln(1+B_T)$.

Droplet evaporation rate is often calculated using the classical model, which is based on heat and mass transfer analogy [3]: $g_\mathrm{v} = 2\pi \cdot \rho_\mathrm{vg} \cdot D_\mathrm{vg} \cdot R \cdot Sh_\mathrm{f} \cdot \ln(1+B_\mathrm{M})$, in which a modified Sherwood number is used: $Sh_\mathrm{f} = 2 + F_\mathrm{M}^{-1} \cdot (Sh-2)$; $F_\mathrm{M} = (1+B_\mathrm{M})^{0.7} B_\mathrm{M}^{-1} \cdot \ln(1+B_\mathrm{M})$. The Hertz–Knudsen–Langmuir Kinetical model [5] including the “Universal” evaporation approximation [8] and the Shorin–Kuzikovsky model, based on the Stefan logarithm [9,10], are also used for determination of evaporation rate.

Connecting the classical evaporation model and the Abramzon–Sirignano model according to the method [3] and additionally evaluating the peculiarities of spectral radiant absorption in semitransparent liquids, heat and mass transfer of water droplets were thoroughly examined in [11]; heating and evaporation of *n*-decane and diesel droplets under conditions of their convective and radiant heating were modelled in [12]. Heat and mass transfer during combustion of sprayed fuel were modelled in [13,14] and other studies. When modelling phase transformation under complicated conditions of droplet heating, it is very important to evaluate close relationship between heat and mass transfer processes in gas and in semitransparent liquid and to take into account the peculiarities of variation of unsteady temperature field, formed in rapidly warming droplets. The methods of combined analytical and numerical research of the "droplet" problem are being applied more and more often [15–19]. The main point of those methods is as follows: the schemes of numerical computation of an unsteady temperature field of a droplet are constructed not for the primary system of nonlinear differential and integral equations, but for its solution – the system of nonlinear algebraic and integral equations. The main advantages of this method are avoidance of the indeterminacies, related to selection of time and coordinate steps and reliable control of numerical research convergence. Two trends of research can be outlined here. In the first case, when modelling heating and evaporation of liquid fuel droplets [16,18], influence of radiation on unsteady temperature field of the droplet is neglected and the asymptotical solutions are used, the numerical schemes of which would not require the algorithm of iterative calculation of the droplet surface temperature. In such case significant savings of computational time are achieved, which is extremely important in CFD (Computational fluid dynamic) codes. In other case the goal is to evaluate the interaction of unsteady transfer processes as thoroughly as possible [15,17,19] and the algorithm of iterative calculation of the droplet surface temperature is necessary. The scheme of surface temperature's calculation is constructed on the basis of the condition of energy flow's balance on the droplet surface: $\vec{q}^{\,+}_{\Sigma}(\tau)+\vec{q}^{\,-}_{\Sigma}(\tau)+\vec{q}^{\,+}_{f}(\tau)=0$. Temperature, which satisfies the above-mentioned condition, is selected using the method of the fastest descent, and then the function $T_R(\tau)$, which defines the change of this temperature, is determined. This function also defines phase transformations: while its value is less than the dew point, liquid vapour (which is present in gas) condenses on the droplet surface; when it is higher, evaporation of liquid occurs. As the function $T_R\ (\tau)$ is determined, the parameters of droplet heat and mass transfer intensity: $q_{\Sigma}(r,\tau), q^{-}_{\Sigma}(\tau), q^{+}_{\Sigma}(\tau), q^{+}_{f}(\tau), m^{+}_{\mathrm{v}}(\tau), R(\tau), T(r,\tau)$ are calculated unambiguously.

In order to evaluate the particular influence of the above factors on heat and mass transfer intensity in sprayed liquid systems, it is necessary to perform a consistent and systematic comparative analysis of heat and mass transfer interaction. For that it is necessary to have a thoroughly researched the simplest case of heat and mass transfer of a droplet. Then it is possible to consequently expand boundary conditions of heat and mass transfer towards more complicated, evaluating more and more factors, simultaneously taking into account individual impact of each case. It is possible to guess that the case, in which a single droplet moving along with a gas flow in the same speed, as it is heated only by conduction, could be the basic case for research of the regularities of the droplet's thermal state change and heat and mass transfer. When modelling heating of large water droplets with initial temperature $T_{l,0}$ in dry air with temperature T_{g} [19], the change of thermal state of the droplets during unsteady evaporation mode is presented by the characteristic curves $T_{R,\mathrm{ch}}(Fo)\equiv f_{Fo}(T_{R,0},\ T_{\mathrm{g}})$, showing the change of surface temperatures of an infinitesimal set of various size droplets, as time scale, expressed by Fourier number, is used. It is verified that it is very efficient to norm the functions of the droplet surface temperature $T_{R,\mathrm{ch}}(Fo)$ according to sprayed water temperature and equilibrium evaporation temperature, as heat supplied to the droplet, evaporates it. In such case the change of non-dimensional temperature

$$\overline{T}_R=\frac{T_R-T_{R,0}}{T_{R,\mathrm{e},k}-T_{R,0}} \tag{1}$$

in the time scale, expressed by the ratio of Fourier numbers Fo/Fo_{ch}, can be presented using the universal curve $T_{R,n}(Fo)=f_{T_R}(Fo/Fo_{\mathrm{ch}})$ [19], which actually joins together all the characteristic curves representing the change of thermal states of droplets, evaporating in air with temperature T_{g}. The values of Fourier numbers, showing unsteady evaporation duration $Fo\equiv Fo_{\mathrm{ch}}=Fo_{\mathrm{e},k}$ and phase transformation duration $Fo\equiv Fo_{\mathrm{ch}}=Fo_{\mathrm{f},k}$ can be called as the characteristic Fourier number (in the case of conductive heating of droplets).

In the case of conductive heating of droplets, equilibrium evaporation temperature of water is defined by the function of gas temperature and partial water pressure in gas $T_{R,\mathrm{e},k}\equiv f_{T_{R,\mathrm{e},k}}(T_{\mathrm{g}},p_{\mathrm{v},\infty})$. The influence of the change of gas temperature and liquid partial pressure in gas on characteristic curves, which represent surface temperatures of evaporating droplets, is not evaluated in the study [19]. The influence of liquid nature is not clear as well. Present study will try to answer these questions.

2. Research method

The change of a droplet mass is defined by liquid evaporation or liquid vapour condensation on its surface

$$\frac{\partial M_l(\tau)}{\partial\tau}=-g_{\mathrm{v}}(\tau). \tag{2}$$

The rate of phase transformations $g_{\mathrm{v}}=4\pi\cdot R^2\cdot m^{+}_{\mathrm{v}}$ is defined by the flow rate of liquid vapour on a droplet surface. It can be characterized by the expression, using the Shorin–Kuzikovsky model [10]

$$m^{+}_{\mathrm{v}}(\tau)=\frac{D_{\mathrm{vg}}(\tau)\mu_{\mathrm{v}}}{T_{R,\mathrm{vg}}(\tau)R_{\mu}R(\tau)}\left\{p_{\mathrm{v},R}(\tau)-p_{\mathrm{v},\infty}(\tau)+\frac{\mu_{\mathrm{v}}}{\mu_{\mathrm{vg}}(\tau)}\right.$$
$$\left.\times\left[p\cdot\ln\frac{p-p_{\mathrm{v},\infty}(\tau)}{p-p_{\mathrm{v},R}(\tau)}-p_{\mathrm{v},R}(\tau)+p_{\mathrm{v},\infty}(\tau)\right]\right\}. \tag{3}$$

In the case of large droplets temperature and pressure drops in the Knudsen layer can be neglected; hence temperature of vapour and gas mixture in the vicinity of a droplet is close to the droplet surface temperature: $T_{R,\mathrm{vg}}(\tau) \equiv T_R(\tau)$ and liquid vapour pressure on the droplet surface corresponds to pressure of saturated vapour, selected according to the droplet surface temperature: $p_{\mathrm{v},R}(\tau) \equiv p_{\mathrm{s}}[T_R(\tau)]$.

In the case, when a droplet moves together with a non-radiating gas flow at the same velocities, its surface temperature fulfils the following condition of energy flow's instant balance on the droplet surface

$$q_{\mathrm{c}}^{+}(\tau) + q_{\mathrm{c}}^{-}(\tau) - q_{\mathrm{f}}^{+}(\tau) = 0. \tag{4}$$

Energy flux for phase transformations depends on liquid nature and on vapour flux on a droplet surface

$$q_{\mathrm{f}}^{+}(\tau) = m_{\mathrm{v}}^{+}(\tau) \cdot L(\tau). \tag{5}$$

The direction of vapour flow depends on the ratio of partial pressures of liquid vapour in the gas flow and in the vicinity of a droplet $p_{\mathrm{v},\infty}/p_{\mathrm{v},R}$. As $p_{\mathrm{v},\infty}/p_{\mathrm{v},R} > 1$, liquid vapour, present in gas, condenses on the droplet surface. As $p_{\mathrm{v},\infty}/p_{\mathrm{v},\ R} < 1$, liquid evaporates. These peculiarities of phase transformations are perfectly evaluated by the Stefan logarithm in the Shorin–Kuzikovsky model: when $p_{\mathrm{v},\infty}/p_{\mathrm{v},R} > 1$, it is negative, when $p_{\mathrm{v},\infty}/p_{\mathrm{v},R} < 1$ – positive and gains zero value, as $p_{\mathrm{v},\infty}/p_{\mathrm{v},R} \equiv 1$.

The direction of an external convective flow between gas and a droplet is unambiguously influenced by the temperature difference between gas and the droplet; the intensity of the convective flow corresponds to the conductive heat flux

$$q_{\mathrm{c}}^{+}(\tau) \equiv q_{k}^{+}(\tau) = \frac{\lambda_{\mathrm{vg}}(\tau)}{R(\tau)} \cdot [T_{\mathrm{g}} - T_R(\tau)] \cdot f[B_T(\tau)]. \tag{6}$$

Thermal conductivity of saturated vapour and gas mixture is selected according to temperature $T_{\mathrm{vg}} \equiv T_R + x \cdot (T_{\mathrm{g}} + T_R)$. The rules $x \equiv$ "1/2" and $x \equiv$ "1/3" are most popular in this case. The Spalding transfer number is defined by the following expression [15]:

$$B_T(\tau) = \frac{c_{p,\mathrm{vg}}(\tau) \cdot [T_{\mathrm{g}} - T_R(\tau)]}{L(\tau)} \cdot \left[1 + \frac{q_{\mathrm{c}}^{-}(\tau)}{q_{\mathrm{c}}^{+}(\tau)}\right]. \tag{7}$$

During unsteady evaporation the ratio of heat fluxes $q_{\mathrm{c}}^{-}/q_{\mathrm{c}}^{+} < 0$ shows that part of heat, supplied to the droplet from outside, heats it. Under equilibrium evaporation conditions we have $q_{\mathrm{c}}^{-}/q_{\mathrm{c}}^{+} \equiv 0$, provided that internal energy of a cooling droplet does not participate in phase transformations and the influence of radiant flow is neglected.

Convective heat flux inside a droplet is calculated using a modified Fourier law

$$q_{\mathrm{c}}^{-}(\tau) = -\lambda_{\mathrm{ef}}(\tau) \cdot \left.\frac{\partial T(r,\tau)}{\partial r}\right|_{r \equiv R^{-}}, \tag{8}$$

in which effective thermal conductivity $\lambda_{\mathrm{ef}} \equiv \lambda_{\mathrm{l}} \cdot k_{\mathrm{c}}^{-}$ evaluates the influence of liquid circulation on heat transfer inside a droplet. If droplets do not slip in gas, forced circulation is not present [3]. A numerical research of combined heat transfer in warming water droplets [17] shows that the Archimedean forces, arising in droplets due to their non-isothermality, are insufficient to cause natural circulation of liquid inside a droplet. A premise is made that $k_{\mathrm{c}}^{-} \cong 1$, therefore $q_{\mathrm{c}}^{-}(\tau) \equiv q_{k}^{-}(\tau)$. A temperature field in a spherically symmetrical droplet is defined by the model of combined heat transfer in it [15], stating that $q_{\mathrm{r}}(r,\tau) \equiv 0$ in a droplet:

$$T(r,\tau) = T_R(\tau) + \frac{2}{r}\sum_{n=1}^{\infty}\left\{\frac{(-1)^n}{n\pi}\sin\left[n\pi\frac{r}{R(\tau)}\right] \cdot \int_0^{\tau} R(\tau')\frac{\mathrm{d}T_R(\tau')}{\mathrm{d}\tau'} \cdot \exp\left[-a_{\mathrm{l}}(\tau')\left(\frac{n\pi}{R}\right)^2(\tau-\tau')\right]\mathrm{d}\tau'\right\}. \tag{9}$$

Then the gradient of unsteady temperature field in the Eq. (8) is defined by the infinite sum of integral equations

$$\left.\frac{\partial T(r,\tau)}{\partial r}\right|_{r \equiv R^{-}} = \sum_{n=1}^{\infty}\left\{(-1)^n \cdot \frac{2\pi n}{R(\tau)} \cdot \int_0^{\tau} R(\tau')\frac{\mathrm{d}T_R(\tau')}{\mathrm{d}\tau'} \cdot \exp\left[-a_{\mathrm{l}}(\tau')\left(\frac{n\pi}{R}\right)^2(\tau-\tau')\right]\mathrm{d}\tau'\right\}, \tag{10}$$

in which the rate of the droplet temperature change is unambiguously defined, as Eq. (4) is being solved according to an iterative algorithm. The gradient of unsteady temperature field unambiguously defines the direction of thermal conductive flow in a droplet. As the droplet evaporates in a flow of non-radiating gas, liquid will be heated only by the convective flow, supplied to the droplet. Therefore only a temperature field with positive gradient can be formed in the droplet and q_{c}^{-}, calculated by Eq. (8), will be negative during entire duration of phase transformations of the droplet.

The formal equation of the energy flow balance (4) is specified according to the system of Eqs. (3), (5)–(10). In order to make its numerical solution scheme, a droplet is divided into J number of control cross-sections, defined by the universal droplet coordinate $\eta = r/R$ in such a way, that

$$\left.\frac{r_j}{R}\right|_{j \equiv J} \equiv \eta_J = \sum_{j=2}^{J}\Delta\eta_j = \sum_{j=2}^{J}(\eta_j - \eta_{j-1}) = 1. \tag{11}$$

Speculative duration of a droplet Fo_{f} is divided into I number of control time instants to fulfil the condition:

$$\left.\frac{Fo_i}{\bar{F}_{\mathrm{f}}}\right|_{i \equiv I} \equiv \bar{F}o_I = \sum_{i=2}^{I}\Delta\bar{F}o_i = \sum_{i=2}^{I}(\bar{F}o_i - \bar{F}o_{i-1}) = 1. \tag{12}$$

The advantages of iterative calculation of the function $T_R(Fo)$ are used: in the kth iteration for required, but yet unknown values of heat and mass transfer parameters HMT the values, calculated in earlier iterations, are denoted: $\mathrm{HMT}_{i,k} \equiv \mathrm{HMT}_{i,k-1}$, then the parameters $\mathrm{HMT}_{i,k}$ are specified according to calculated temperature $T_{R,i,k}$. To ensure the stability of the numerical scheme for temperature $T_{R,i}$ an exclusion is made for a droplet radius: $R_{i,k} \equiv R_{i-1}$.

For a spherically symmetrical volume of liquid Eq. (2) is presented as follows:

$$\rho_1(\tau)\frac{\partial R^3(\tau)}{\partial\tau} = R^3(\tau)\frac{\partial\rho_1(\tau)}{\partial\tau} - 3R^2(\tau)\cdot m_{\mathrm{v}}^{+}(\tau). \quad (13)$$

Then, evaluating vapour flow density on the droplet surface and volumetric expansion of a warming droplet, the change of its volume is calculated by this scheme

$$R^3_{i,k} = R^3_{i-1} - 3\left[\frac{m^+_{\mathrm{v},i,k}+m^+_{\mathrm{v},i-1}}{\rho_{1,i,k}+\rho_{1,i-1}}\overline{R}^2_{i,k} + \frac{\overline{R}^3_{i,k}}{1.5(\rho_{1,m,i,k}+\rho_{1,m,i-1})}\frac{\rho_{1,m,i,k}-\rho_{1,m,i-1}}{\tau_i-\tau_{i-1}}\right]\cdot(\tau_i-\tau_{i-1}), \quad (14)$$

as the control time index of phase transformation duration $I_{\mathrm{f}} \equiv i$ is being consistently changed in the interval $i \equiv 2 \div I$, until the droplet evaporates out completely. In the expression (14): $\overline{R}_{i,k} \equiv (R_{i-1} + R_{i,k-1})/2$.

Temperature of a warming droplet surface is selected using the method of the fastest descent, requiring accuracy of the balance (4) to be not less than 0.01%. Proper control of time step ensures the change of mean mass temperature of a warming droplet to be not higher than 1 K.

3. Results and discussion

The change of thermal state and phase transformation intensity of water, *n*-hexane, *n*-decane and *n*-heptane droplets is numerically modelled, as the droplets are carried without slipping by air flow with temperature T_{g}. It is stated that small amount of liquid is sprayed, therefore air temperature and composition of gas mixture does not change due to warming and evaporation of droplets: $T_{\mathrm{g}}(\tau) = \mathrm{const}$ and $p_{\mathrm{v},\infty}(\tau) = \mathrm{const}$. Droplets do not slip in gas, therefore $Re(\tau) = 0$ and they are heated only by conduction. The Gerry empirical equation was applied for definition of water saturated vapour on the droplet surface [20]; in the case of liquid hydrocarbons the Antoine's approximation was used [21].

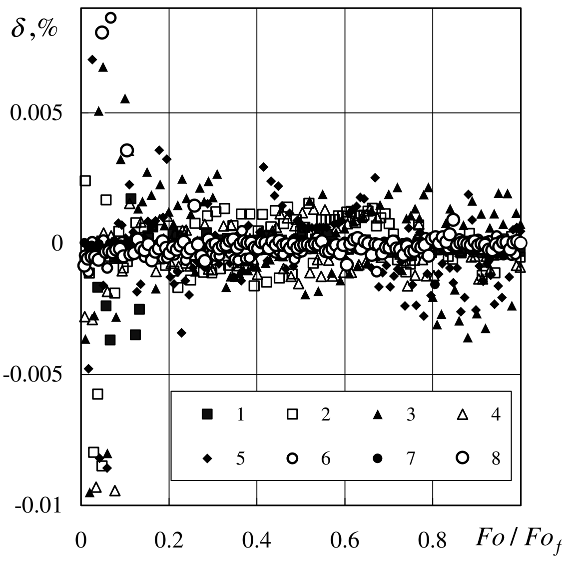

Fig. 1. Control of disbalance of energy flows on a droplet surface $\delta = \{1 - [q_{\mathrm{c}}^{+}/(q_{\mathrm{f}}^{+} - q_{\mathrm{c}}^{-})]\}\cdot 100\%$. Liquid: (1–5) water, (6) *n*-hexane, (7) *n*-heptane, (8) *n*-decane; T_{g}, K: (1–3, 8) 750, (4–6) 500, (7) 1000; $T_{1,0}$, K: (1–3, 7, 8) 300, (4–6) 275; $p_{\mathrm{v},\infty}/p$: (1, 6–8) 0, (2, 4) .2, (3, 5) .4; $R = .00005$ m.

According to [22–24], disbalance of energy flows on a droplet surface was strictly controlled during the entire numerical experiment (Fig. 1). Huge influence of droplet surface temperature on energy flows showed up (Fig. 2). When selecting the instant value of droplet surface temperature by the method of the fastest descent, it is necessary to evaluate the change of this temperature by 0.001° (Fig. 2a). Then high reliability of energy flow balance is ensured (Fig. 2b). Actually it is reached after making 13 iterations. That shows a reliable operation of the iterative scheme. Requirement of the condition (4) was well ensured during entire lifetime of the droplet. A bit higher variation of disbalance was detected only during the initial stage of unsteady phase transformations (Fig. 1). It is probably

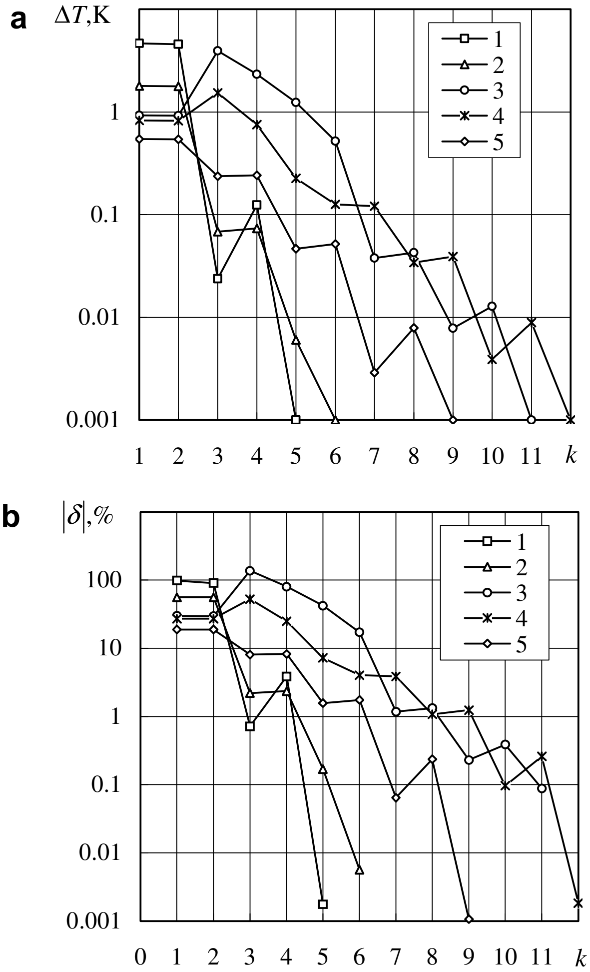

Fig. 2. Selection of a droplet surface temperature by the method of the fastest descent (a) and its influence on disbalance of energy flows on a droplet surface (b). Duration of droplet heating *Fo*: (1) .013, (2) .052, (3) .15, (4) .18, (5) .28; $\Delta T \equiv |T_{R,\mathrm{K}} - T_{R,k}| + 0.001$; $p_{\mathrm{v},\infty}/p = .3$; $T_{\mathrm{g}} = 750$ K; $T_{1,0} = 300$ K.

connected to rapid growth of droplet surface temperature and existence of liquid vapour in gas. As the mode of liquid vapour condensation on the droplet surface comes to end, the moment of change of phase transformation nature approaches, when the vector of vapour flow density changes its direction and its module gains zero value. All modelled cases of heat and mass transfer passed control, similar to Fig. 1.

3.1. Thermal state of conductively heated evaporating droplets

Thermal state of droplet varies during unsteady phase transformation process. The unsteady temperature field has characteristic dynamics in the droplets of different liquids (Fig. 3). Different liquid droplets with similar initial temperature $T_{1,0}$, heated conductively, during unsteady evaporation mode warm up to equilibrium evaporation temperature $T_{1,e,k}$, which significantly depends on liquid nature. As mentioned earlier, this temperature also depends on the parameters of the gas flow. The influences of gas temperature T_g and partial pressure of liquid in gas $p_{v,\infty}$ are essential.

One can notice qualitatively common tendencies of thermal state change of evaporating droplets. When they are heated by conduction, the sign of unsteady temperature field's gradient can be only positive $T(r,\tau)$. Variation of this temperature field is very active during the mode of unsteady phase transformations, though during equilibrium evaporation it is close to isothermal and independent of size of the droplets. At the beginning surface layers of the droplets warm rapidly, later the rate of their warming decreases and, as central layers of the droplets warm rapidly, non-isothermality of the droplets decreases. During equilibrium evaporation, as $Fo \geqslant Fo_e$, we have $T(r,Fo) \equiv T_{R,e,k}$.

When defining thermal state of a warming droplet, the following functions of characteristic temperature variation are important: surface temperature $T_R(Fo)$, centre temperature $T_C(Fo)$ and mean mass temperature of the droplet $T_{1,m}(Fo)$. The rate of change of the latter temperature indicates the intensity of heat flow, which warms the droplet

$$q_1(\tau) \equiv -\frac{1}{3}\rho_1(\tau)c_{p,1}(\tau)R(\tau)\frac{\partial T_{1,m}(\tau)}{\partial \tau}. \tag{15}$$

Surface temperature of droplets is important for heat and mass transfer between droplets and gas flow; temperature difference between droplet centre and surface defines non-isothermality of the droplet, which can cause natural circulation inside it.

If gas contains liquid vapour, it can condense on cold surface of the droplets for a while. The duration of condensation is defined by time instant $Fo \equiv Fo_{k0}$, when change of phase transformations occurs from condensation mode to evaporation mode. More liquid vapour is in gas, smaller temperature difference is between saturation temperature and temperature of evaporating sprayed liquid (Fig. 4).

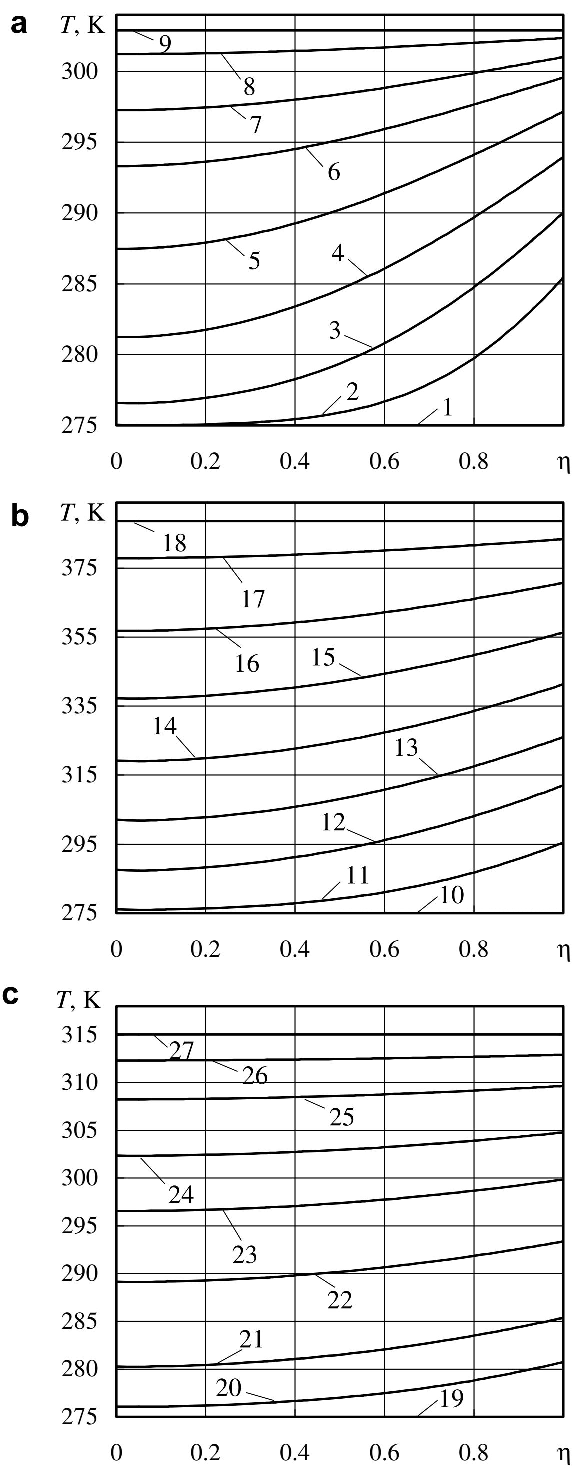

Fig. 3. Unsteady temperature field in *n*-hexane (a), *n*-decane (b) and water (c) droplets. Duration of droplet heating *Fo*: (1, 10, 19) 0, (9, 18, 27) $Fo_{e,k}$, (2) .0486, (3) .0971, (4) .162, (5) .243, (6) .34, (7) .437, (8) .631, (11) .0859, (12) .2, (13) .315, (14) .458, (15) .63, (16) .859, (17) 1.26, (20) .144, (21) .289, (22) .578, (23) .867, (24) 1.156, (25) 1.59, (26) 2.17; $T_g = 500$ K; $T_{1,0} = 275$ K; $p_{v,\infty}/p = 0$; $R = .000025$ m.

Hence, it is possible to construct a consistent scheme of phase transformation on a droplet surface, which contains condensation mode, as well as unsteady and equilibrium evaporation modes

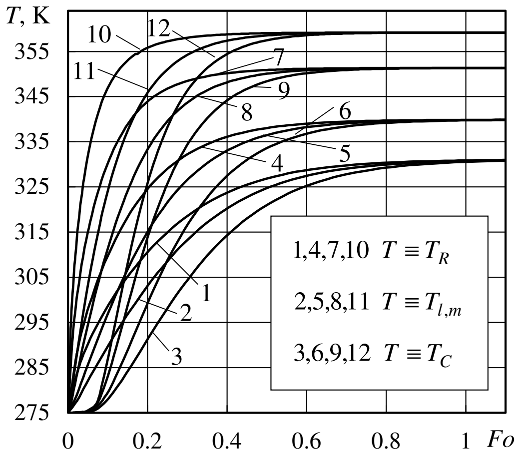

Fig. 4. Influence of air humidity on the change of water droplet temperature. $p_{v,\infty}/p$: (1–3) 0, (4–6) .1, (7–9) .3, (10–12) .5; $T_g = 1000$ K; $T_{l,0} = 275$ K; $R = .0001$ m.

$$Fo \equiv 0 \div Fo_{k0} \div Fo_e \div Fo_f. \tag{16}$$

If droplets evaporate in dry gas or if temperature of sprayed liquid is higher than dew point, $Fo_{k0} = 0$. The end of the condensation mode is unambiguously defined by time instant, when vapour flow density passes through its zero value $m_v^+(Fo \equiv Fo_{k0}) \equiv 0$ (Fig. 5). It is quite difficult to predict the time instant, when equilibrium evaporation begins. It is a matter of agreement. In this study we state, that liquid evaporation is equilibrium from the moment, as $Fo \equiv Fo_e$ meets the following requirement:

$$|T_{l,m,e} - T_{l,m}(Fo > Fo_e)| < 0.01\text{ K}. \tag{17}$$

In the case of a single droplet: $T_g(\tau) = \text{const}$ and $p_{v,\infty}(\tau) = \text{const}$, therefore droplet temperature remains constant during equilibrium evaporation: $T_{l,e,k}(\tau) = \text{const}$. All droplets warm up to this temperature, independently of their size.

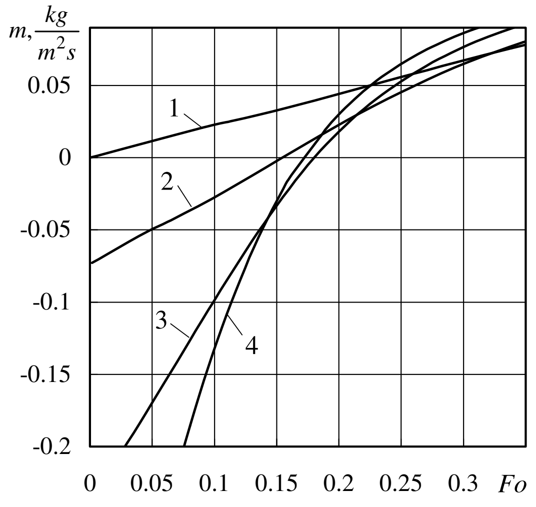

Fig. 5. The change of nature of phase transformation, as water droplets warm in humid air. $p_{v,\infty}/p$: (1) 0, (2) .1, (3) .3, (4) .5; $T_g = 1000$ K; $T_{l,0} = 275$ K; $R = .0001$ m.

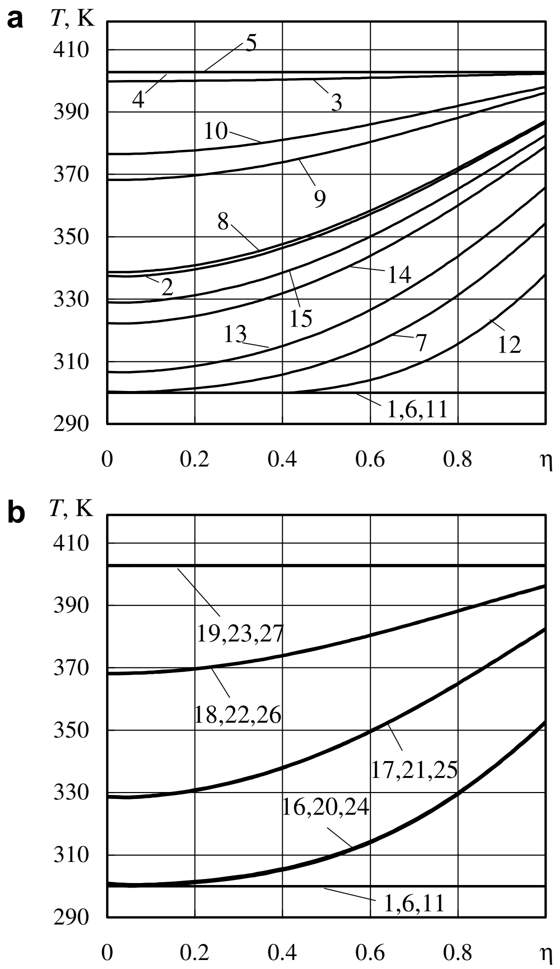

Fig. 6. Warming of *n*-decane droplets in real time (a) and Fourier number based (b) scales. $R \times 10^{-6}$, m: (1–5, 16–19) 30, (6–10, 20–23) 50, (11–15, 24–27) 70; Warming time τ, s: (1, 6, 11) 0, (2, 7, 12) .0026, (3, 8, 13) .0074, (4, 9, 14) .0111, (5, 10, 15) .0125; Fourier number Fo: (1, 6, 11) 0, (16, 20, 24) .085, (17, 21, 25) .2, (18, 22, 26) .36, (19, 23, 27) 1.12; $T_g = 750$ K; $T_{l,0} = 300$ K.

Fig. 6a shows the temperature field of *n*-decane droplets of various diameters after the same warming time; while Fig. 6b indicates various warming durations, corresponding to similar values of Fourier number.

Smaller droplets have already reached equilibrium evaporation conditions (Fig. 6a, curves 4 and 5); bigger droplets are still warming intensively (Fig. 6a, curves 10 and 15). Though the change of temperature field in droplets and the duration of unsteady phase transformations significantly depend on size of droplets (Fig. 6a), dynamics of warming of droplets of various diameters is identical in the time scale, expressed by Fourier number (Fig. 6b). This conclusion is confirmed in all modelled cases of thermal state change of different – size droplets, as boundary conditions regarding sprayed liquid temperature and gas parameters are identical for droplets of various sizes, but of the same liquid. The change of characteristic centre, surface and mean mass temperatures of the unsteady temperature field is distinctive for water (Fig. 7a) and *n*-heptane

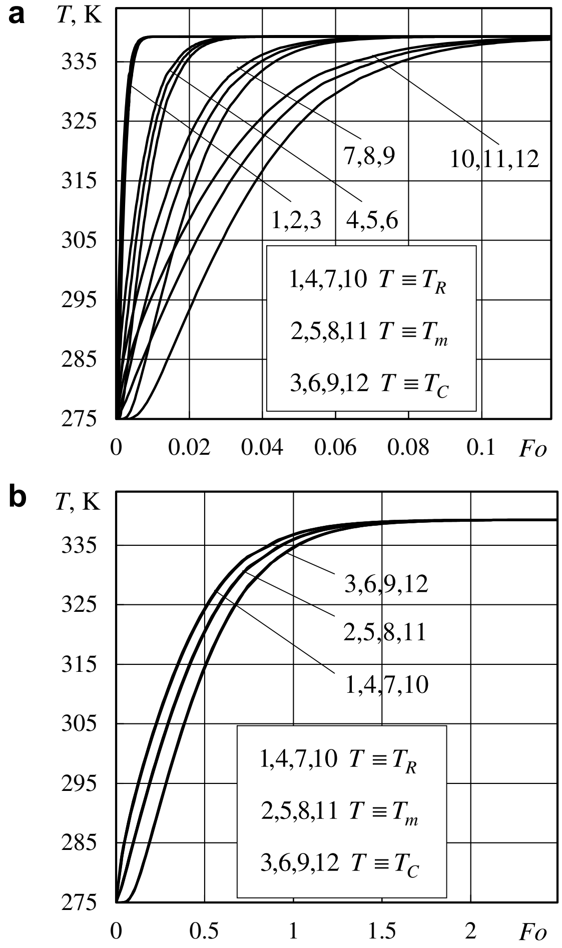

Fig. 7. Variation of characteristic temperatures of an unsteady temperature field in water droplets in real time (a) and Fourier number (b) scales. $R \times 10^{-6}$, m: (1–3) 25, (4–6) 50, (7–9) 75, (10–12) 100; $T_g = 500$ K; $p_{v,\infty}/p = .2$; $T_{l,0} = 275$ K.

droplets (Fig. 8a) of different size. Though variation of the curves in Fourier number based scale is independent of droplet size (Figs. 7b and 8b). Hence, in Fourier number based scale the characteristic curves indicate warming dynamics of infinite set of large droplets. This important conclusion allowed optimizing the whole numerical research.

The peculiarities of boundary conditions of heat and mass transfer can influence dynamics of the characteristic curves, which show the change of evaporating and warming droplet's thermal state. Temperature of sprayed liquid, gas temperature and partial pressure of liquid in gas are the essential factors. In order to evaluate the change of the thermal state of sprayed liquid from the same position, it is purposeful to norm the characteristic curves, which describe dynamics of mean mass temperature (Fig. 9a) in regards to the initial and equilibrium evaporation temperatures of sprayed liquid (Fig. 9b)

$$\overline{T}_m(Fo) = \frac{T_{l,m}(Fo) - T_{R,0}}{T_{l,e,k} - T_{R,0}} \tag{18}$$

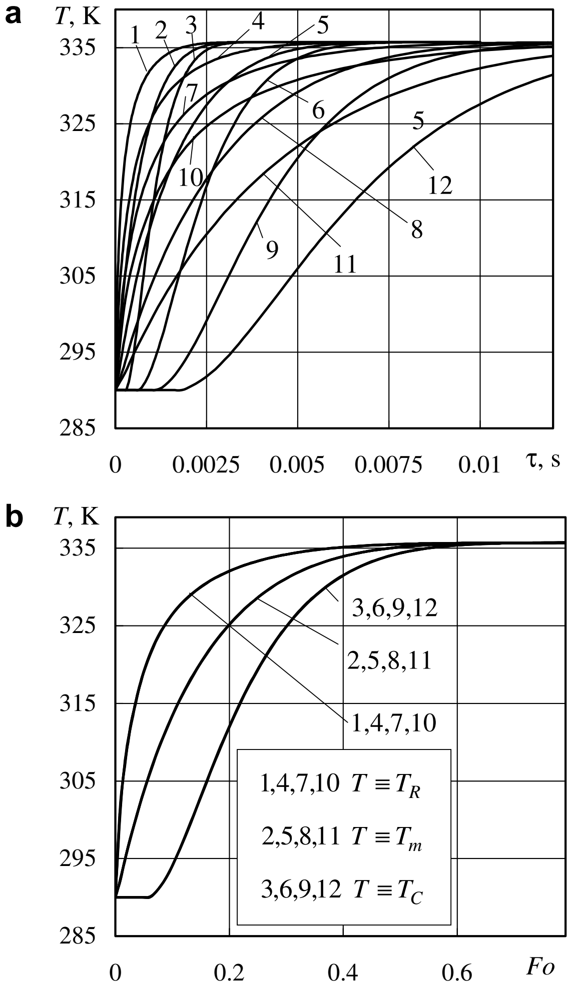

Fig. 8. Variation of characteristic temperatures of an unsteady temperature field in *n*-heptane droplets in real time (a) and Fourier number (b) scales. $R \times 10^{-6}$, m: (1–3) 20, (4–6) 30, (7–9) 40, (10–12) 50; $T_g = 750$ K; $p_{v,\infty}/p = 0$; $T_{l,0} = 290$ K.

and to present them in the universal scale of evaporation time $0 \div 1$, expressed by the ratio of Fourier numbers $\overline{Fo}_e = Fo/Fo_{e,k}$. The results of the performed numerical research prove that the characteristic curves $\overline{T}_m(\overline{Fo}_e)$ do not depend on temperature of sprayed liquid (Fig. 9c). That offers an idea, that maybe it is possible to obtain the universal curve of the change of droplet's state change, which could unite all characteristic curves into one with allowable accuracy. For that it is necessary to evaluate not only liquid temperature's influence, but also influence of gas temperature and vapour pressure in it. According to the research results (Fig. 10) of *n*-hexane droplet warming and evaporation in gas at various temperatures it is possible to state that the characteristic curves $\overline{T}_m(\overline{Fo}_e)$ are not sensible to gas temperature (Fig. 10b), though one can detect stratification of the curves $\overline{T}_m(Fo)$ according to temperature T_g (Fig. 10a).

Presence of liquid vapour in gas is an important factor in the case of phase transformations of sprayed water. In the case of liquid fuel evaporation chemical reactions

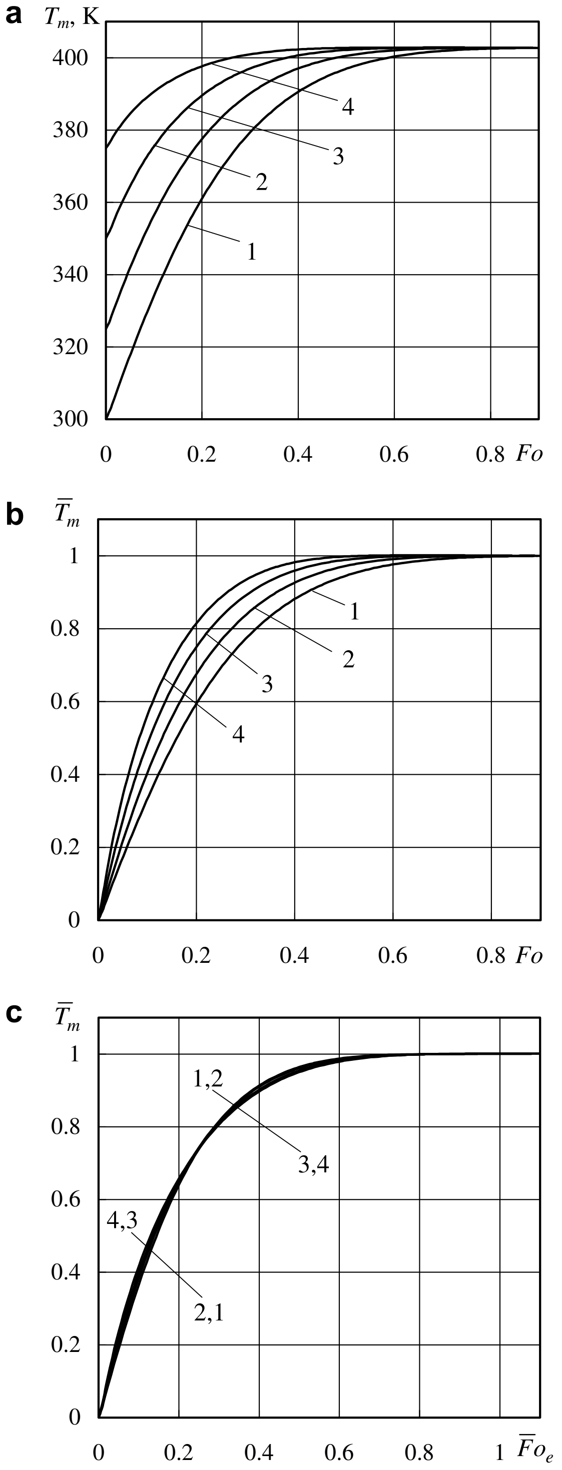

Fig. 9. Dependence of the characteristic curves of droplet mean mass temperature on temperature of sprayed *n*-decane $T_{l,0}$, K: (1) 300, (2) 325, (3) 350, (4) 375; Duration of unsteady evaporation $Fo_{e,k}$: (1) 1.11, (2) .94, (3) .78, (4) .65; $T_g = 750$ K; $p_{v,\infty}/p = 0$.

occur, therefore it is possible to neglect existence of liquid vapour in gas. Though equilibrium evaporation temperature of sprayed liquid significantly depends on air humidity (Fig. 11a) and the curves $\overline{T}_m(Fo)$ are also sensible to it

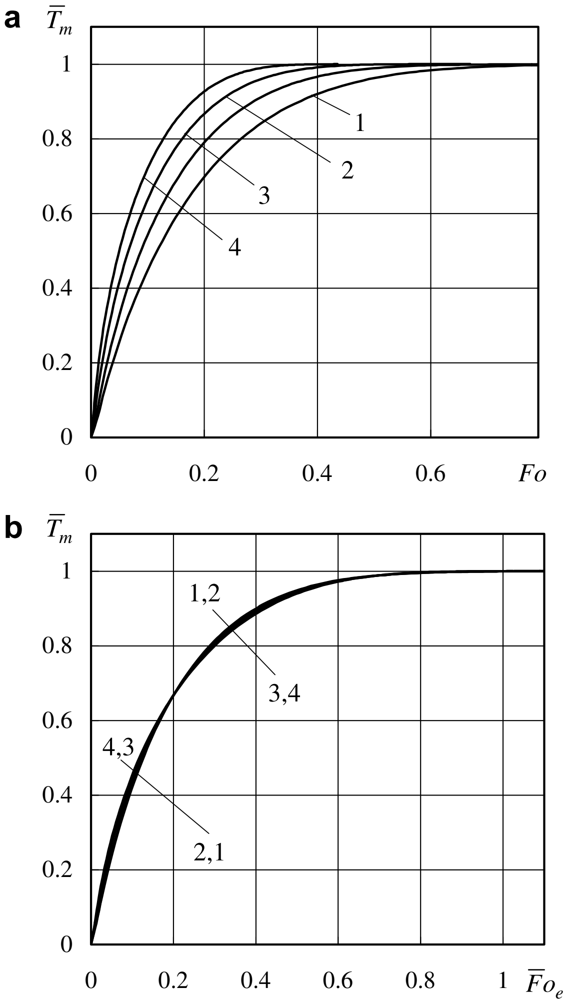

Fig. 10. Dependence of the characteristic curves of *n*-hexane droplet mean mass temperature on gas temperature T_g, K: (1) 500, (2) 600, (3) 750, (4) 1000; Equilibrium evaporation temperature $T_{l,e,k}$, K: (1) 302.9, (2) 306.85, (3) 309.62, (4) 311.14; Duration of unsteady evaporation $Fo_{e,k}$: (1) .92, (2) .71, (3) .55, (4) .42; $T_{l,0} = 300$ K; $p_{v,\infty}/p = 0$.

(Fig. 11b), air humidity practically has no influence on the characteristic curves $\overline{T}_m(\overline{Fo}_e)$ (Fig. 11c).

The influence of liquid nature on the characteristic curves $\overline{T}_m(\overline{Fo}_e)$ was also examined (Fig. 12). The characteristic curves of thermal state change of hydrocarbon droplets under various conditions of heat and mass transfer well enough correlate with the experimental research results (Fig. 13). The correlation of numerical research results of water droplets with experimental research results is presented in [17,19].

Liquid nature influences droplet warming rate and its equilibrium evaporation temperature (Fig. 13a). The change of its mean mass temperature's curves $\overline{T}_m(Fo)$ is also distinctive, but liquid nature does not cause essential change of the characteristic curves $\overline{T}_m(\overline{Fo}_e)$, though they have negligible differences (Fig. 12c).

The performed analysis of warming and evaporation process of various liquids allows stating that the thermal

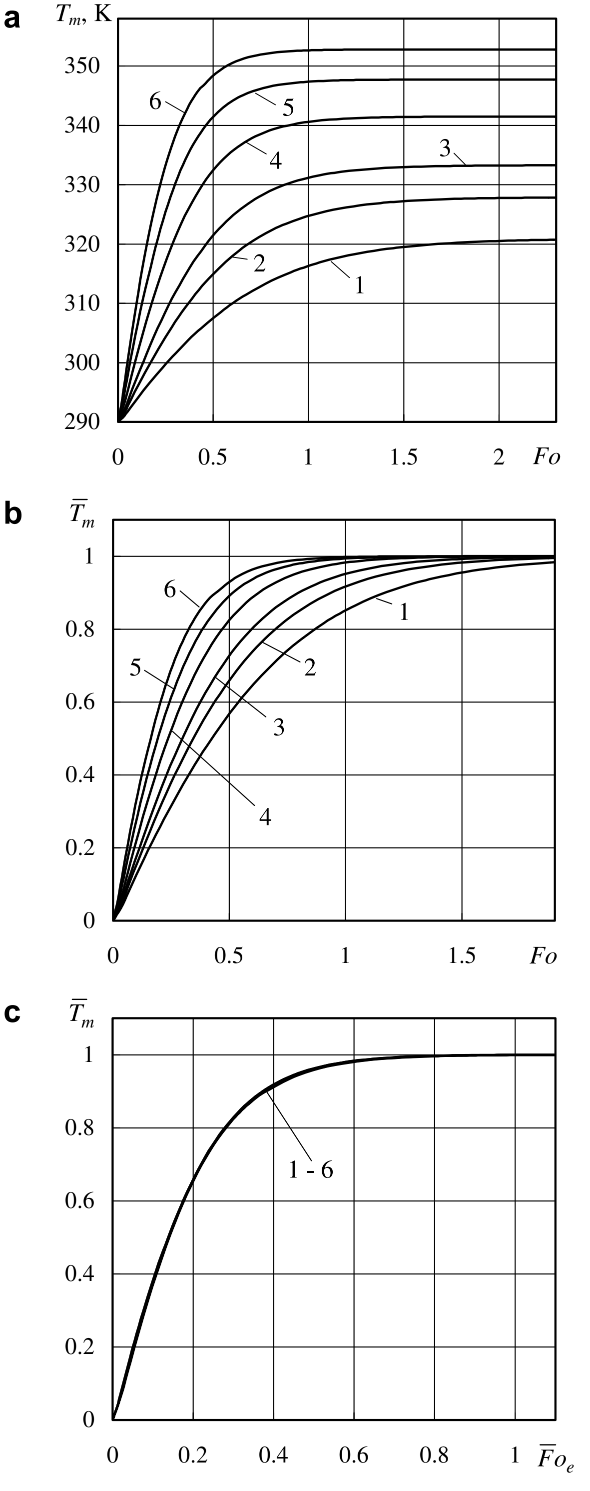

Fig. 11. Dependence of the characteristic curves of water droplet mean mass temperature on air humidity $p_{v,\infty}/p$: (1) 0, (2) .05, (3) .1, (4) .2, (5) .3, (6) .4; Equilibrium evaporation temperature $T_{l,e,k}$, K: (1) 320.89, (2) 327.88, (3) 333.28 (4) 341.49, (5) 347.73 (6) 352.8; Duration of unsteady evaporation $Fo_{e,k}$: (1) 3.08, (2) 2.49, (3) 2.14, (4) 1.68, (5) 1.37, (6) 1.16; $T_g = 600$ K.

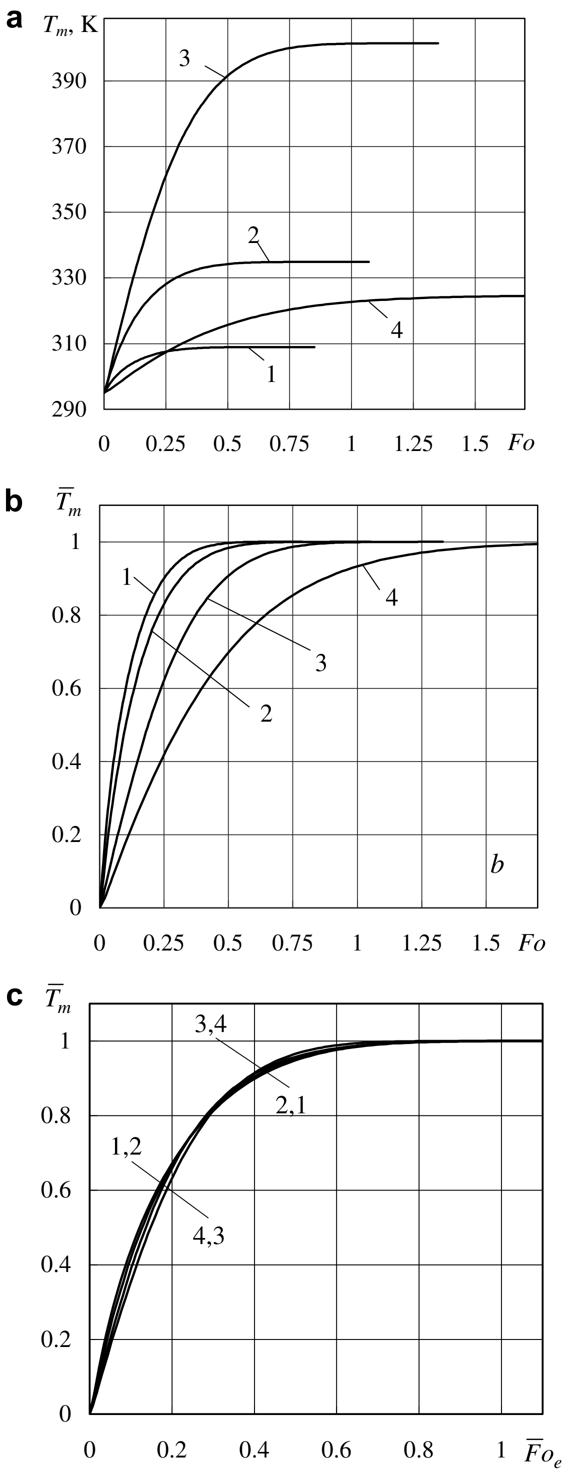

Fig. 12. Influence of liquid nature on the characteristic curves of droplet mean mass temperature: (1) *n*-hexane, (2) *n*-heptane (3) *n*-decane, (4) water; Equilibrium evaporation temperature $T_{l,e,k}$, K: (1) 308.95, (2) 334.89 (3) 401.44, (4) 324.7; Duration of unsteady evaporation $Fo_{e,k}$: (1) .62, (2) .8, (3) 1.28, (4) 2.28; $T_g = 700$ K, $p_{v,\infty}/p = 0$.

state of sprayed liquid droplets, as they are heated by conduction, changes in qualitatively similar way. All modelled cases of heat and mass transfer gave the result, that the change of the thermal state of sprayed liquid droplets can be described by a universal normalized droplet mean mass temperature (Fig. 14). During unsteady phase transforma-

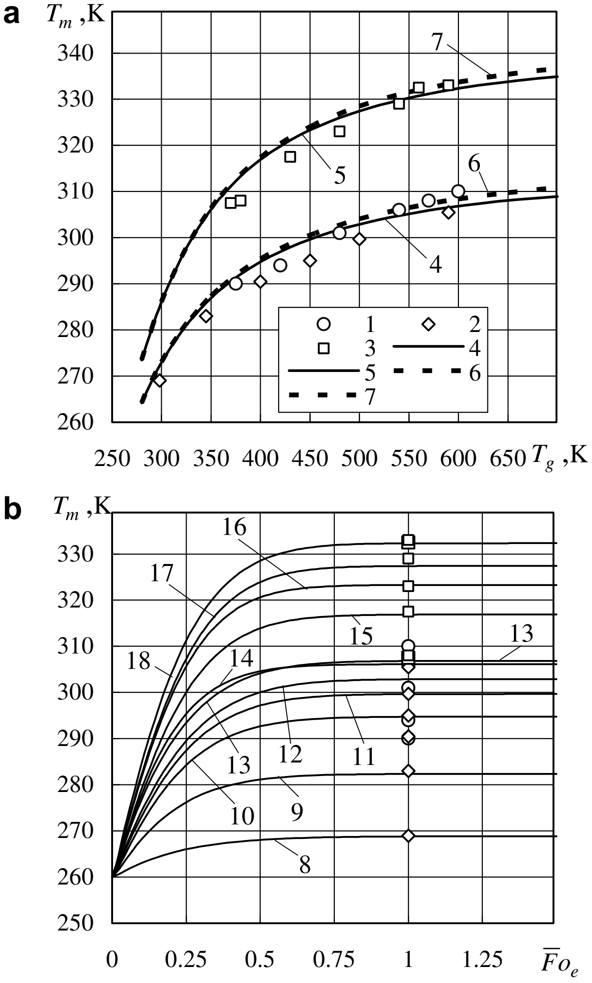

Fig. 13. Equilibrium evaporation temperature (a) and warming dynamics (b) of *n*-hexane (1, 2, 4, 6, 8–13) and *n*-heptane (3, 5, 14–18) droplets: points – experiment [4]: (1, 3) El. Wakil et al., (2) C.G. Downing; lines – numerical research: (6, 7) $x \equiv$ "1/3", (4, 5, 8–18) $x \equiv$ "1/2", as air temperature T_g, K: (8) 290, (9) 330, (10, 15) 400, (11, 16) 450, (12, 17) 500, (13, 18) 600, (14) 350; $T_{1,0} = 260$ K; $p_{v,\infty}/p = 0$.

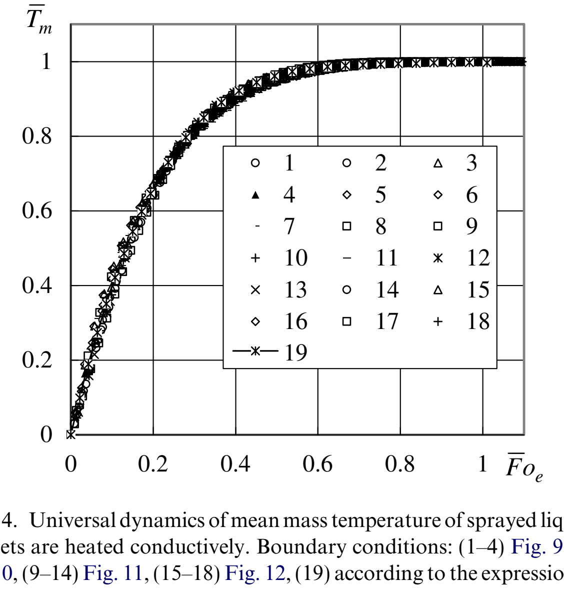

Fig. 14. Universal dynamics of mean mass temperature of sprayed liquid, as droplets are heated conductively. Boundary conditions: (1–4) Fig. 9, (5–8) Fig. 10, (9–14) Fig. 11, (15–18) Fig. 12, (19) according to the expression (19).

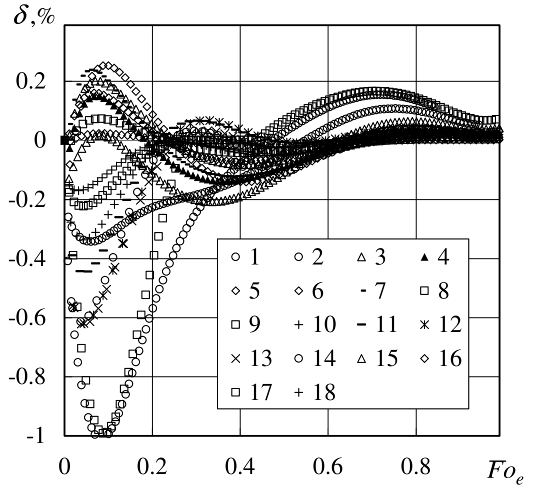

Fig. 15. Reliability of calculation of warming droplet mean mass temperature according to the expression (19). Boundary conditions correspond to Fig. 14. $\delta = (1 - T_{m,1\text{-}18}/T_m) \cdot 100\%$; $T_{m,1\text{-}18} \equiv T_{l,0} + \overline{T}_{m,1\text{-}18} \cdot (T_{m,e} - T_{1,0})$.

tion mode the latter temperature can be defined by the empirical expression of the universal curve $\overline{T}_m(\overline{Fo}_e)$

$$\overline{T}_{m,1\text{-}18}(\overline{Fo}_e) = 4.78\overline{Fo}_e - 8.725\overline{Fo}_e^2 + 7.126\overline{Fo}_e^3 - 2.184\overline{Fo}_e^4. \quad (19)$$

This expression ensures calculation of mean mass temperature of sprayed liquid by the accuracy not less than ±1% (Fig. 15), as the droplets are heated by conduction. Mean mass temperature of warming droplets are calculated with accuracy of ±0.3% in most cases of boundary conditions of the numerical research. The accuracy of the expression (19) is a bit lower only under the conditions of significantly unsteady evaporation of *n*-decane droplets.

It is obvious that in order to express dynamics of thermal state change of warming droplets, described by the expression (19), using the change of the droplet mean mass temperature, it is necessary have the functions $Fo_{e,k} \equiv f_{1,Fo}(T_{1,0}, T_g, \bar{p}_v)$ and $T_{1,e,k} \equiv f_{1,T}(T_g, \bar{p}_v)$, which describe duration of unsteady phase transformation $Fo_{e,k}$ and equilibrium evaporation temperature $T_{1,e,k}$, respectively. Actually, the above-mentioned functions can be specified only by complex research of the "droplet" problem, using numerical modelling methods. A dilemma of methodology selection appears in this situation. It is necessary to evaluate the peculiarities of the task, take into account all the premises and accuracy of the obtained research results. Convective heating intensity of motionless evaporating droplets and unsteady temperature field can be regarded as the essential. Under more complicated boundary conditions of the "droplet" problem droplet slip in gas, radiant absorption in semitransparent liquid and quantity of droplets are the factors that become important. Their influence can be evaluated according to deviation of the functions Fo_e, $T_{1,e}$ and $T_1(Fo)$ from $Fo_{1,e,k}$, $T_{1,e,k}$ and $T_{1,k}(Fo)$. Despite of the variety of the methods to solve the "droplet" problem, convective heating intensity of an evaporating droplet is mostly often calculated by the methods, based on the similarity theory. In this case one can find some uncertainties related to selection of a characteristic temperature (“1/2” or “1/3” rules), various expressions of the Spalding heat transfer number’s $f(B_T)$ correction in the empirical expressions of convective heat flow and various evaporation models. The rule “1/2” or “1/3”, according to which characteristic temperature of gas and vapour mixture is selected, becomes more and more important for determination of the value of equilibrium evaporation temperature, as gas temperature grows (Fig. 13a) and makes sensible influence on the change of thermal state of droplet (Fig. 16), independently of the applied method of convective heating intensity calculation (Fig. 16b) and which evaporation model is used (Fig. 16a). During the examined cases of the combinations of convective heating and evaporation models universality of the expression (19) was verified (Fig. 16c), though peculiar values of $Fo_{1,e,k}$ and $T_{1,e,k}$ were obtained.

3.2. Phase transformations of droplets

Number of factors influences the changes of droplet geometry. During the stage of unsteady phase transformations droplets expand, as liquid intensively warms. This volumetric expansion can be intensified by liquid vapour condensation on droplet surface or weakened by liquid evaporation. Hence, it is quite difficult to predict the changes of droplet geometry at the initial stage of unsteady evaporation. The boundary conditions of heat and mass transfer and nature of sprayed liquid decide which effect actually prevails. Presumable influence of volumetric expansion can be evaluated by use of a hypothetical droplet model, which predicts existence of a mass source, equivalent to phase transformation rate: $g_{\text{sor}}(Fo) \equiv 4\pi R^2(Fo) \cdot m_{\text{v}}^{+}(Fo)$. Then phase transformation on a warming droplet surface has no influence on its mass, therefore during unsteady evaporation process the change of the droplet volume depends on the change of liquid thermal state

$$\frac{R_{\text{e}}^3}{R_0^3} \equiv \frac{\rho_{1,0}(T_{R,0})}{\rho_{1,\text{e}}(T_{1,\text{e},k})}. \tag{20}$$

The curves, which show dynamics of a hypothetical droplet volume, calculated in different cases of heat and mass transfer, suitably satisfy the condition (20) (Fig. 17).

Volumetric expansion for different liquids is different, because each liquid has characteristic thermal state of droplet equilibrium evaporation, though initial temperatures are the same. Higher is equilibrium evaporation temperature of liquid, more intense is the effect of volumetric expansion process. In the case of examined liquids this effect is mostly vivid for n-decane (Fig. 17, curve 12).

Temperature change of warming liquids depends not only on liquid volumetric expansion, but also on the rate

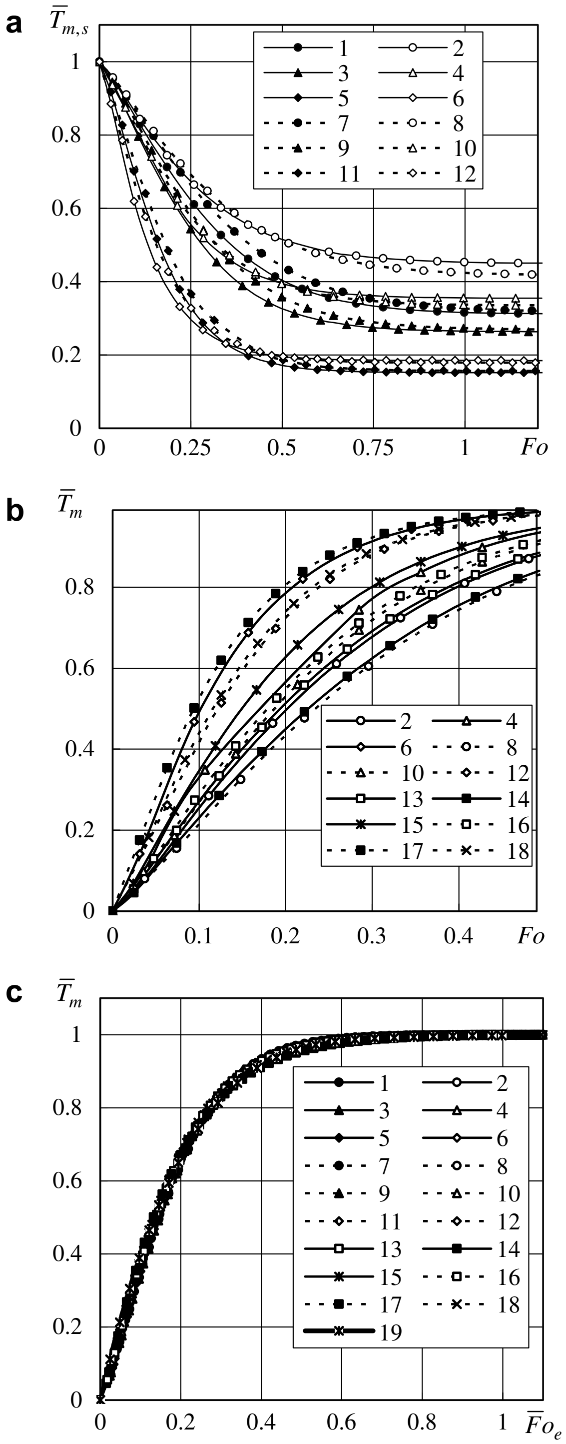

Fig. 16. Influence of evaporation model (a) and heating model (b) on dynamics of droplet warming and on the universal curve of thermal state change (c). Evaporation model: (1, 3, 5, 7, 9, 11) classical, (2, 4, 6, 8, 10, 12–18) Shorin–Kuzikovsky; convective heating model: (1–12) Abramzon–Sirignano and classical, (13–18) Experimental correlation; continuous lines $x \equiv$ “1/2”; dotted lines $x \equiv$ “1/3”; $T_{1,e,k}$, K: (1) 343.91, (2) 331.09, (3) 348.38 (4) 339.9, (5) 358.72 (6) 355.61, (7) 343.36, (8) 334.15, (9) 347.82 (10) 341.75, (11) 358.27 (12) 356.15, (13) 329.97, (14) 333.05, (15) 339.14, (16) 340.97, (17) 355.25, (18) 355.79; $Fo_{e,k}$: (1) 1.52, (2) 1.44, (3) 1.33, (4) 1.19, (5).94, (6).76, (7) 1.75, (8) 1.66, (9) 1.48 (10) 1.35, (11) 1.06 (12).87, (13) 1.4, (14) 1.62, (15) 1.1, (16) 1.3, (17) .71, (18) .86; $p_{\text{v},\infty}/p$: (1, 2, 7, 8, 13, 14) 0, (3, 4, 9, 10, 15, 16) .1, (5, 6, 11, 12, 17, 18) .4; $\overline{T}_{m,s} \equiv (T_s - T_{1,m})/(T_s - T_{1,0})$; $T_g = 1000$ K.

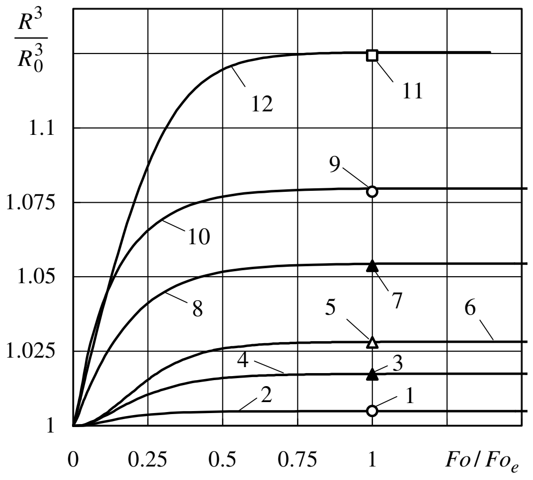

Fig. 17. Volumetric expansion of a hypothetical droplet: (1–6) water, (7, 8) *n*-hexane, (9, 10) *n*-heptane, (11, 12) *n*-decane; dots – condition (20); T_g, K: (1, 2) 400, (3, 4) 1500, (5, 6–8, 11, 12) 500, (9, 10) 1000; $p_{v,\infty}/p$: (1–4, 7–12) 0, (5, 6).4.

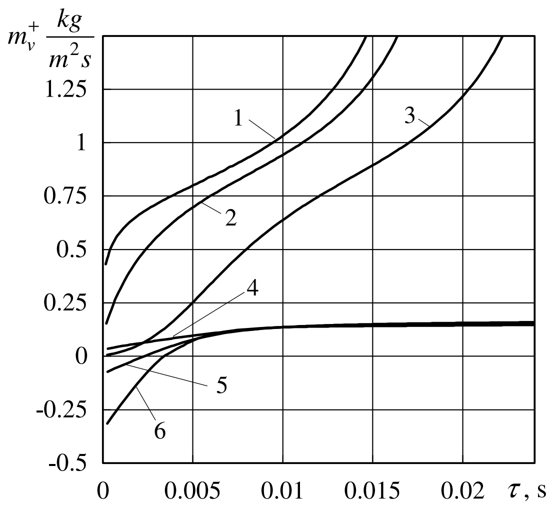

Fig. 18. Influence of liquid nature and air humidity on phase transformation intensity: (1) *n*-hexane, (2) *n*-heptane (3) *n*-decane, (4–6) water; Equilibrium evaporation temperature $T_{l,e,k}$, K: (1) 308.95, (2) 334.89 (3) 401.44, (4) 324.7, (5) 335.61, (6) 348.95; Duration of unsteady evaporation $Fo_{e,k}$: (1) .62, (2) .8, (3) 1.28, (4) 2.28, (5) 1.66, (6) 1.15; Intensity of phase transformations at the instant of equilibrium evaporation beginning $m^{+}_{v,e}$: (1) 1.4, (2) 1.38, (3) 2.6, (4) .16, (5) .15, (6) .14; Partial pressure of water vapour in air $p_{v,\infty}/p$: (1–4) 0, (5) .1, (6) .3; $T_g = 700$ K; $R_0 = 0.00004$ m.

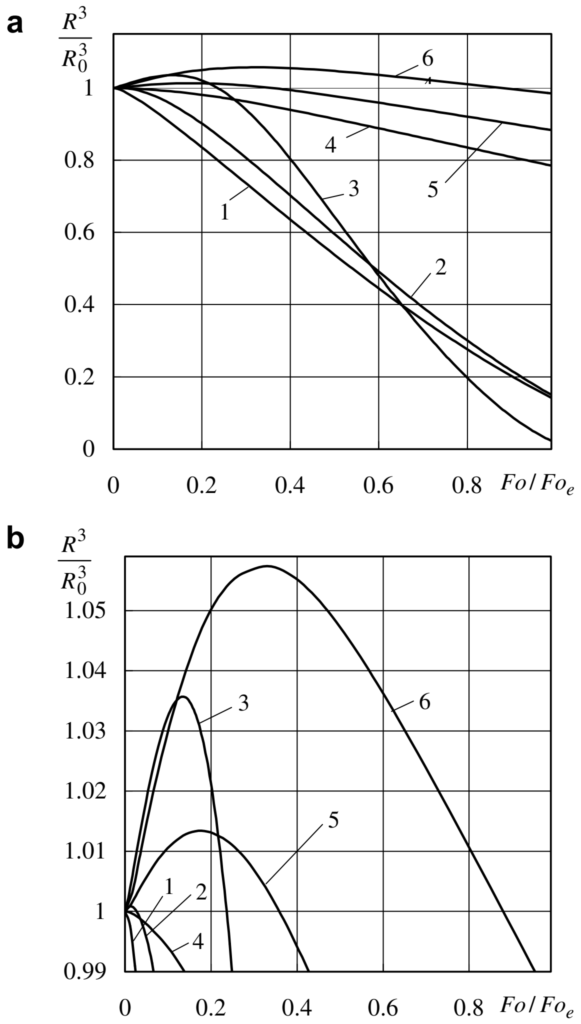

Fig. 19. Influence of liquid nature and vapour condensation process on volumetric expansion of droplets during unsteady phase transformation mode. Markings the same as in Fig. 18.

of liquid evaporation (Fig. 18). Liquid vapour condensation on droplet surface is also important (Fig. 19).

The main factors, which decide phase transformations of sprayed liquid during unsteady evaporation mode, are (a) liquid nature and spraying dispersity – they decide evaporation rate and volumetric expansion during warming process; (b) initial temperature of liquid – makes preconditions for condensation mode to appear; (c) gas parameters – decide the intensity of external energetic influence on droplets. Therefore the change of phase transformation's intensity (Fig. 18) and volume (Fig. 19) are characteristic in different boundary conditions of heat and mass transfer. Despite of that, the change of a droplet dimension $\overline{R}$ and intensity of phase transformations $\bar{m}$, expressed in Fourier number based time scale are defined by the characteristic curves, identical for all large droplets of the same liquid (Figs. 20–22).

Equilibrium evaporation temperature of *n*-hexane is the lowest, comparing with other examined liquids, but evaporation rate of *n*-hexane droplets is the highest, therefore volumetric expansion of *n*-hexane droplets does not evoke any peak in the curve that represents droplet volume variation (Fig. 19, curve 1). The curve, which shows variation of volume of water droplets, evaporating in dry air, also has no extreme point. A slight increment of volume is present for *n*-heptane droplets, while volume of *n*-decane droplets increases more than in 3% due to volumetric expansion. As air humidity increases, the period of condensation occupies bigger and bigger part of unsteady phase transformation mode; condensation of liquid vapour on droplet surface intensifies; the droplets warm quicker and

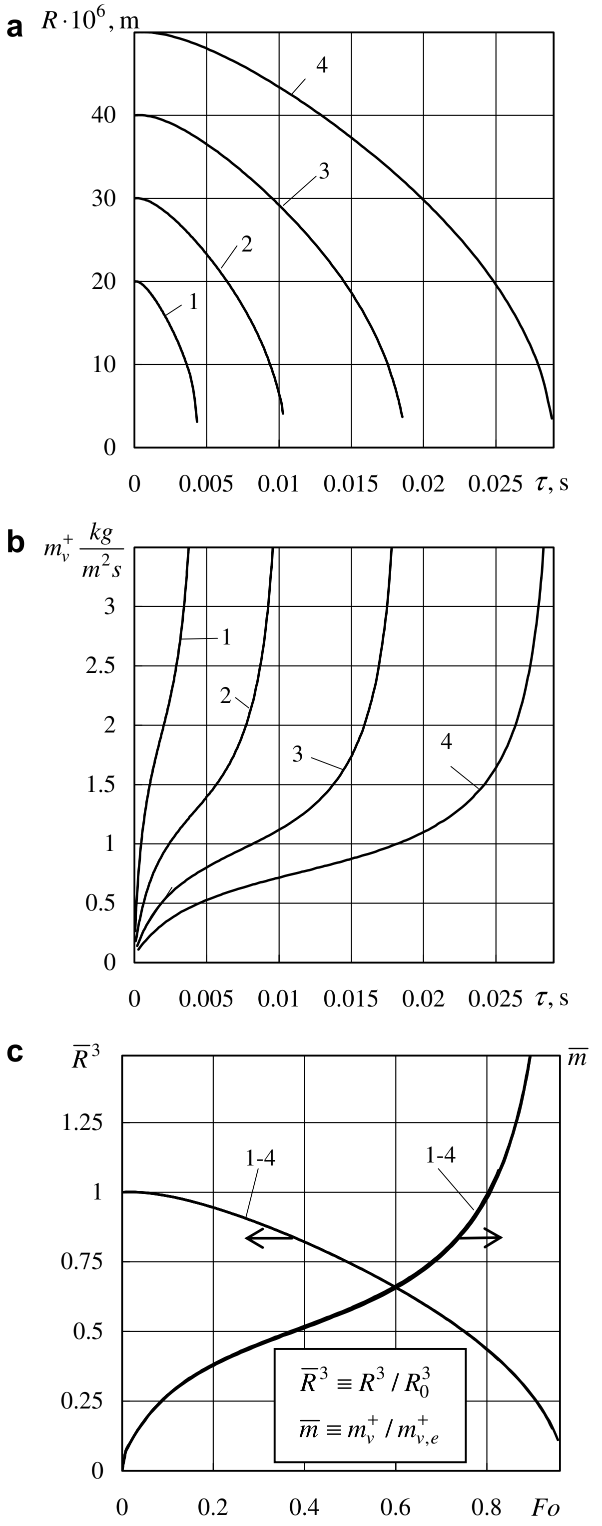

Fig. 20. Variation of the parameters of *n*-heptane droplet's physical transformations in real time (a, b) and Fourier number (c) based scales. $R \times 10^{-6}$, m: (1) 20, (2) 30, (3) 40, (4) 50; $T_g = 750$ K; $T_{l,0} = 290$ K.

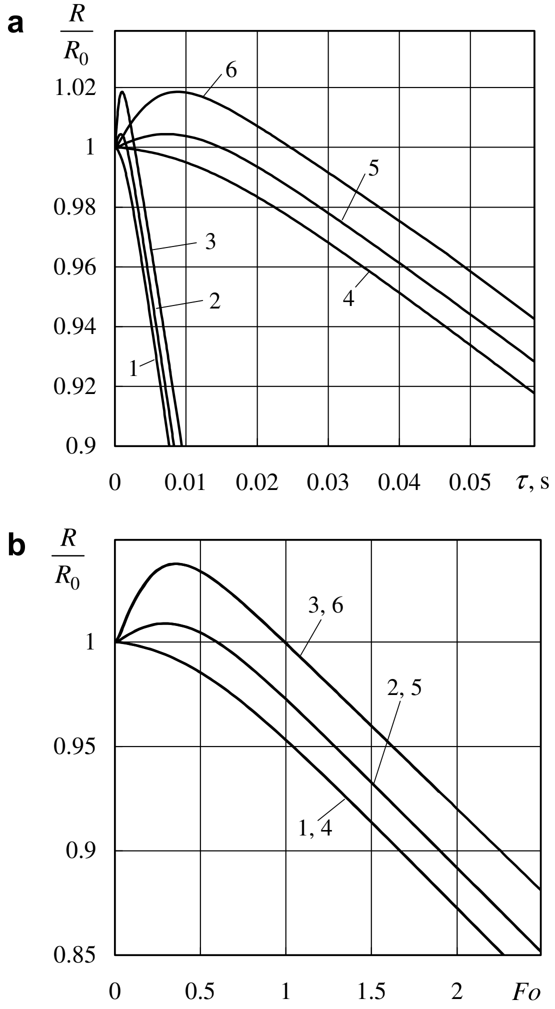

Fig. 21. Variation of water droplet's dimension in real time (a) and Fourier number (b) based scales. Partial pressure of water vapour in air $p_{v,\infty}/p$: (1, 4) 0, (2, 5).1, (3, 6).3; $R_0 \times 10^6$, m: (1–3) 20, (4–6) 60; $T_g = 700$ K.

quicker; their volumetric expansion becomes more and more vivid. When $p_{v,\infty}/p > .3$, the size of droplets, which reach equilibrium evaporation mode, is close to their initial size (Fig. 19, curve 6). The evaporation rate of warming hydrocarbons is higher, comparing to water. Unsteady phase transformation covers perceptible part of a droplet lifetime cycle. Hydrocarbon droplets loose more than 80% of their mass during unsteady phase transformation mode. *n*-Decane droplets during this phase loose more than 90%. Obviously, when temperature of sprayed liquid is closer to equilibrium evaporation temperature, the influence of unsteady phase transformation will be smaller. In this case the ratio of sprayed liquid temperature on equilibrium evaporation temperature is an important parameter: closer the ratio $T_{m,0}/T_{m,e,k}$ is to one, weaker is the influence of the unsteady phase transformation mode. Nature of liquid has a crucial influence on the characteristic curves, which represent variation of phase transformation parameters of droplets (Fig. 23).

The characteristic curves are very distinctive during the unsteady phase transformation mode. Relative surface area of droplets varies linearly during the equilibrium evaporation mode and then the universal curves, which represent the physical transformation parameters of all liquids, are

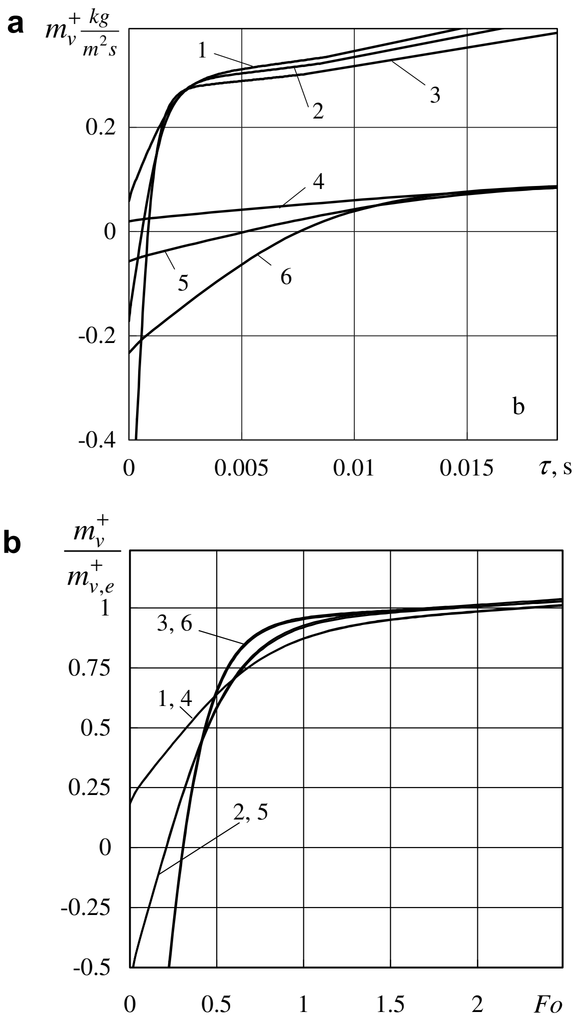

Fig. 22. Variation of the intensity of water droplet’s phase transformation in real time (a) and Fourier number based (b) scales. $m^+_{\mathrm{v,e}}$: (1) 1.4, (2) 1.38, (3) 2.6, (4) .16, (5) .15, (6) .14; Other markings the same as in Fig. 21.

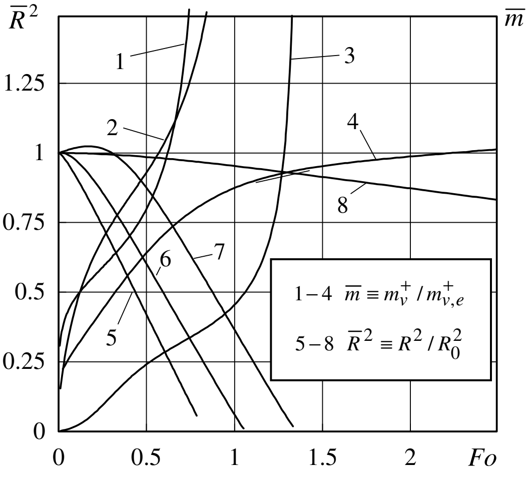

Fig. 23. Influence of liquid’s nature on the characteristic curves, which represent variation of phase transformation parameters of droplets: (1, 5) *n*-hexane, (2, 6) *n*-heptane (3, 7) *n*-decane, (4, 8) water; $p_{\mathrm{v},\infty}/p = 0$; $T_{\mathrm{g}} = 700$ K.

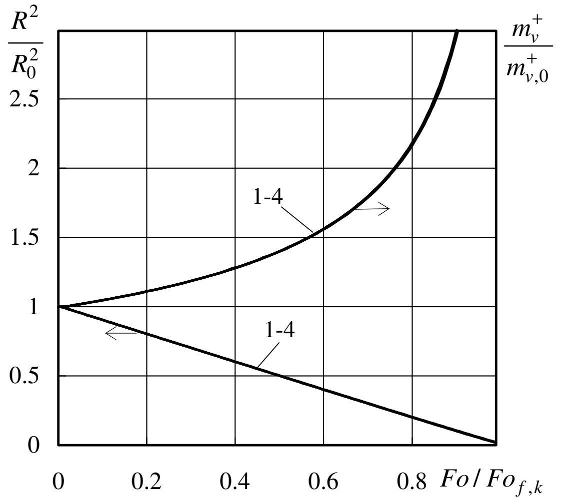

Fig. 24. The universal curves, which represent variation of equilibrium evaporation parameters, as droplets are heated by conduction: (1) *n*-hexane, (2) *n*-heptane (3) *n*-decane, (4) water. $T_{m,0}/T_{m,\mathrm{e},k} \equiv 1$.

similar (Fig. 24). Taking into account plenty of the factors that influence the peculiarities of unsteady phase transformation, it quite complicated to construct one universal curve of droplet dimension change for all liquids. As distinct from the case of the universal curve of thermal state change, search for this curve can be stimulated not by practical, but more by scientific interest, because for each liquid it would be necessary to know the functions $m^+_{\mathrm{v,e},k} \equiv f_{m^+_{\mathrm{v}}}(T_{m,\mathrm{e}}, R_{m,\mathrm{e}}, T_{\mathrm{g}}, p_{\mathrm{v},\infty})$, which are distinctive and complicated. The main indeterminacy is related to a very specific effect of droplet volumetric expansion, as liquid warms and simultaneously unsteady phase transformation occurs on its surface. Here it is possible to conditionally group liquid droplets into sensitive and insensitive to volumetric expansion and also to exclude the case of possible liquid vapour condensation on droplet surface. Insensibility of a droplet dimension change to liquid volumetric expansion is ensured by the effects of liquid expansion and evaporation, which are equal and therefore compensate each other. Domination of volumetric expansion effect for a while or presence of liquid vapour condensation causes the extreme point in the curves of droplet dimension change. Talking about examined liquids, *n*-decane can represent sensitive liquids; *n*-hexane – insensitive; water, sprayed to humid air, well represents the case, sensitive to liquid vapour condensation (Fig. 19). The ratio of temperatures $T_{m,0}/T_{m,\mathrm{e},k}$ is important to this aspect as well. As $T_{m,0}/T_{m,\mathrm{e},k} \to 1$, droplets of all liquids can be assigned to the group of insensitive to volumetric expansion.

4. Conclusion

The results of the performed numerical research can be handled in two important aspects. The first is related to the development of the methods, which allow evaluating unsteady heat and mass transfer parameters of sprayed liquid systems in relatively simply way. The second aspect is related to a systematic estimation of transfer process interaction in two-phase droplet and gas flows. The offered simple interpretation (19) of the universal curve, representing thermal state change of sprayed liquid allows not only to evaluate warming of droplets during the mode of unsteady phase transformations, but also to avoid the solution of the internal "droplet" problem in engineering calculations, as the change of a droplet volume is calculated according to the scheme (14). It is worth noticing, that the approximation (19) is completely enough in order to evaluate volumetric expansion in the scheme (14), though it is necessary to take into account the change of droplet surface temperature, when estimating the influence of phase transformation. The approximation of the universal curve of the droplet surface temperature would usefully serve for that. The emphasis of the latter approximation did not go into the scope of this work.

If heat and mass transfer conditions are more complicated than the case of large droplet heating, which was modelled in this study, heat and mass regularities in the systems of sprayed liquid will be influenced by a number of additional factors: slipping of droplets in gas, reciprocity of droplets, radiant absorption in semitransparent liquid, influence of the Knudsen layer on heating and evaporation of small droplets, etc. The performed research makes premises for systematic evaluation of those factors.

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